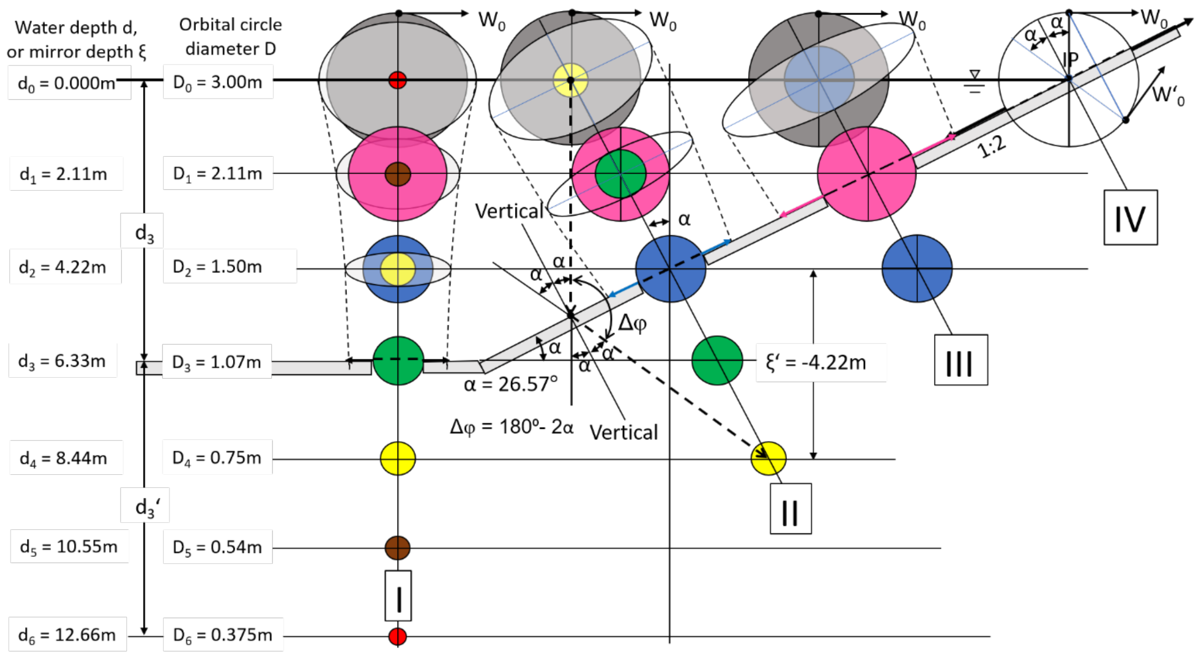
Influence of the Sea-Bed-Inclination on the Orbital Movement in the Area of Decreasing Water Depth – New Theoretical Approach –

The linear wave theory according to Airy-Laplace (1842), which has so far mainly been used by engineers, is on the one hand limited to the assumption of a flat sea bed and on the other hand violates the law of conservation of mass (continuity condition). This is no longer the case with the author's new approach, which involves an extension of the mirroring procedure referred to on level ground by Schulejkin (1956). The extension by the author now refers to ground inclinations $0^o \leq \alpha \leq 90^o$. According to this approach, the deep-water waves characterized by circular orbital movements, interfere with likewise circular orbital movements of smaller diameter, which are assigned to the waves *exponentially reduced reflected* from the inclined sea bottom. Considering the phase jump $\Delta\varphi = \pi - 2\alpha$ (Büsching 2019) between incident and reflected waves, depending on the inclination α of the sea ground, elliptical orbital paths are formed. The long main axes of the ellipses are parallel to the seabed angle of inclination α. With increasing approach to the bottom on the one hand, and to the intersection IP (of the slope inclination with the still water level) on the other, the long axes of the ellipses increase at the expense of the short axes until the latter disappear completely on the bottom. For this purpose, the local long axes assume the double amount of the initial orbital circle diameter and in the wave cycle there is carried out a linearly polarized particle oscillations, cf. figure.



Principle of the elliptical paths of water particles existing in the presence of water waves on an inclined plane, based on the theory of "exponentially reduced reflection" (ERR).

In analogy to the *optical* reflection with equal angles of incidence and reflection α, a vertical incidence beam is assumed here with respect to reflection axis II. It hits the inclined surface of the sea bottom and is reflected from there with the starting angle α related to the vertical. In the negative mirror depth ($\xi' = -4.22$m corresponding to $d_4 = 8.44$m), the orbital circle diameter, according to $D = D_0 \cdot e^{-2\pi\, d/L}$, is reduced to $D_4 = 0.75$m.

Since the horizontal orbital velocity vector W_0 at the incident wave crest is assumed as a reference point, the phase jump Δφ occurs between the incident and the outgoing beam as angle of rotation $\Delta\varphi = 180^0 - 2\alpha$. Superimposing the orbital velocities (by magnitude and direction), assigned to the respective *exponentially reduced* virtual orbital circles, on those of the respective incident circular orbital motions of the initial deep water waves, the result is the represented elliptical orbits rotated by the angle α. Analogous to the optical reflection, it was taken into account that the orbital velocity vectors have opposite directions of rotation on their orbital circles.

The obtained result is shown here in principle for a wave of height H = 3.0m and length L = 38.00m with respect to the given boundary conditions.

The representations of the other parameters related to the dimensions of the associated model investigations can be taken from the original work of Büsching, Fritz (2019): „Vibration Interferences in the Limited Orbital Field of Sea Waves in Theory and Physical Model“: https://doi.org/10.24355/dbbs.084-202002031131-0.

It should be noted:

For the time being, the acceptance of the found results seems to be impaired by the fact that the author used a relatively complex spectral method developed by him for the analysis of irregular waves in his model investigations, cf. his earlier publications. This method has hardly been reproduced by other researchers.

A practically important result, which also reflects the core of the new theory, is the definition of the complex reflection coefficient $\Gamma = C_r\, e^{i\Delta\varphi}$. http://www.digibib.tu-bs.de/?docid=00062890.

This states that the reflection of water waves is not only determined by the ratio of the heights of the reflected to the incident wave ($C_r = H_r / H_i$), but also by the phase shift Δφ (phase jump) between the incident and the reflected wave, which depends on the sea bed inclination α. The author uses the expression

$$C_{r,i} = \frac{\sqrt{E_{\max,i}} - \sqrt{E_{\min,i}}}{\sqrt{E_{\max,i}} + \sqrt{E_{\min,i}}}$$

for the Fourier components (or partial waves) *i* of an energy spectrum of irregular waves. This special feature has apparently not yet been sufficiently understood by other researchers either.

Since the new theory agrees with the continuity condition and since the phase jump Δφ could be reproduced in a physical model, its proof could be considered as a link that was missing until now. If necessary, a future consideration of the phase jump Δφ could mean a paradigm shift in surf research, including the tsunami problem and even with non-linear theories.

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